

**TEST 1R**

**This is the repeat version of TEST 1, which was held during Session.**

**This repeat test should be attempted by those students who missed Test 1, or who wish to improve their mark in Test 1.**

**IF YOU ARE ATTEMPTING TEST 1 (Repeat):**

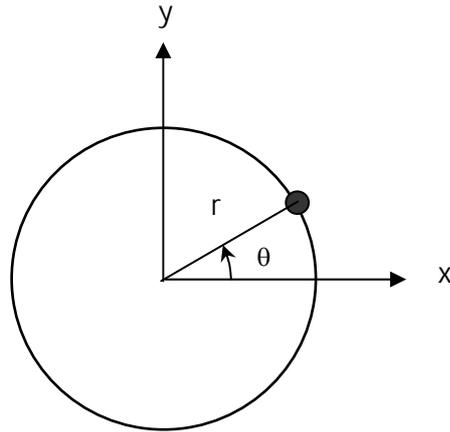
**CROSS THIS BOX**



**AND INDICATE YES ON THE FRONT (TITLE) PAGE.**

**QUESTION 6** [Marks 20]

A particle executes uniform circular motion with radius  $r$ , as shown in the figure.



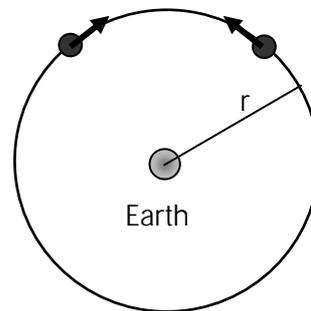
- (a) From the figure, derive an expression for the position vector  $\mathbf{r}$  in terms of  $r$ , the angular velocity  $\omega$ , and the time  $t$  by projecting  $\mathbf{r}$  onto the  $x$  and  $y$  axes. Take the angle  $\theta$  at  $t = 0$  to be zero.
- (b) From (a), derive the velocity  $\mathbf{v}$  and the acceleration  $\mathbf{a}$  of the particle. Express the result for the acceleration in terms of the position vector.
- (c) Calculate the scalar product  $\mathbf{v} \cdot \mathbf{r}$ . What does this say about the relative orientation of  $\mathbf{v}$  and  $\mathbf{r}$ ?

A stone is tied to a string and is swung in a horizontal circle of radius 1.2m at 1.8m above the ground. The string breaks and the stone travels a horizontal distance of 9.0m before striking the ground.

- (d) What was the centripetal acceleration of the stone while it was in circular motion?
- (e) The shadow of a particle undergoing uniform circular motion is projected onto a screen that is perpendicular to the plane of the circle. Demonstrate that the shadow executes simple harmonic motion.

**QUESTION 7** [Marks 20]

A satellite of mass  $m$  is in a circular orbit of radius  $r$  about the Earth with mass  $M_e$  and radius  $R_e$ .



- (a) Derive an expression for the kinetic energy of the satellite in terms of the gravitational constant  $G$ ,  $m$ ,  $M_e$ , and  $r$ .
- (b) Hence, derive an expression for the launch speed of the satellite from Earth required to place it in this orbit. Show that your result reduces to the escape speed from the Earth when  $r \rightarrow \infty$ .

Two satellites are in circular orbits about the Earth at radius  $r$  and are on a collision course, as shown in the figure.

- (c) Derive an expression for the total mechanical energy for the two-satellite-plus-Earth system before the collision in terms of  $G$ ,  $m$ ,  $M_e$ , and  $r$ .

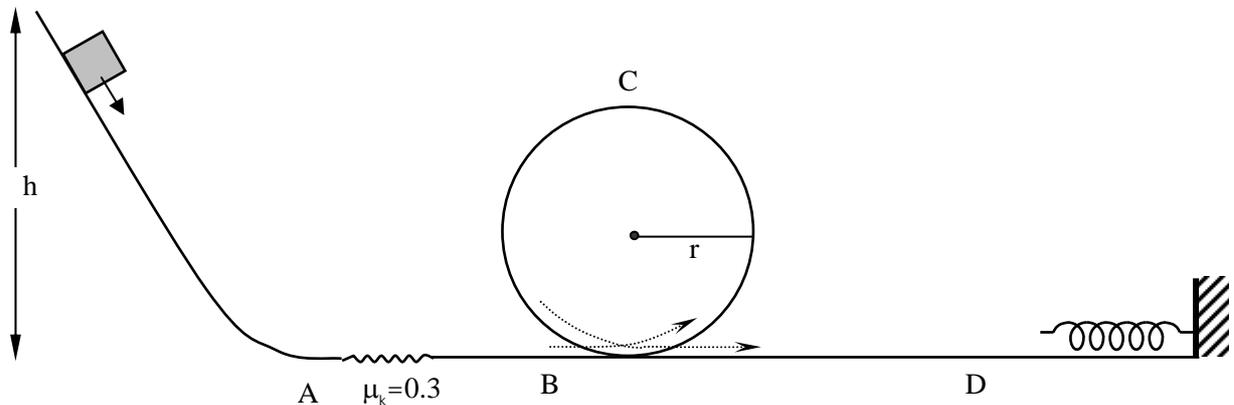
The satellites collide completely inelastically (the wreckage stays in one piece after the collision).

- (d) What is the total mechanical energy of the system after the collision?
- (e) Describe the subsequent motion of the wreckage.

**QUESTION 8**

**[Marks 20]**

A block is released from a height  $h$  on a frictionless incline. At the base of the incline there is a rough surface of length  $\ell$  and the coefficient of kinetic friction between the block and the surface is  $\mu_k = 0.3$ . The surface becomes frictionless again before a circular loop of radius  $r$  which is followed by a spring attached to a wall. The block remains in contact with the surface at all times. The set-up is shown in the figure; different points of the track are labelled A, B, C, D.

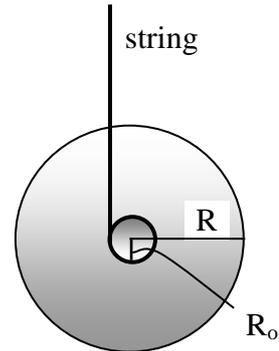


- (a) What is the speed of the block at point A in terms of the height  $h$ ?
- (b) What is the speed of the block at point B in terms of the given parameters?
- (c) At point C the block is at the maximum height inside the loop.
  - (i) What is the speed of the block at this point? Express your answer in terms of the given parameters.
  - (ii) Draw a free-body diagram for the forces acting on the block at point C.
  - (iii) Find the minimum height  $h$  such that the block maintains contact with the loop at point C. Take  $\ell = 3\text{cm}$  and  $r = 5\text{cm}$ .
- (d) The block is brought to rest beyond point D after compressing the spring by  $3.0\text{cm}$ . Determine the value for the spring constant  $k$ , if  $\ell = 3\text{cm}$ ,  $r = 5\text{cm}$ ,  $h = 18\text{cm}$ , and the mass of the block  $m = 0.025\text{kg}$ .

**QUESTION 9**

[Marks 20]

A yo-yo consists of two discs of radius  $R$  connected by a shaft of radius  $R_0$  around which string is wrapped (see figure). The yo-yo has a mass  $m = 0.020\text{kg}$ ,  $R = 3.0\text{cm}$ , and  $R_0 = 0.30\text{cm}$ . Assume that the rotational inertia of the yo-yo about the centre of mass is the same as that of a disc,  $I = mR^2/2$ . The string is held while the yo-yo is released from rest; as the yo-yo falls, the string unwinds without slipping.

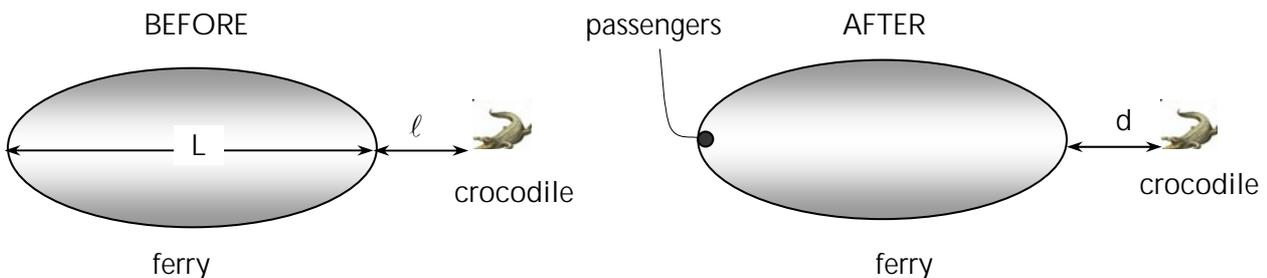


- Draw a free-body diagram for the yo-yo. Which of the forces produces a torque about the centre of mass?
- Write down Newton's second law for the torque about the centre of mass in terms of the force(s).
- Write down Newton's second law for the forces acting in the vertical direction.
- Hence, find expressions in terms of  $m$ ,  $g$  (acceleration due to Earth's gravity),  $R$ , and  $R_0$  for
  - the tension in the string  $T$
  - the linear acceleration  $a$  of the yo-yo.
- At the instant when the string is fully unwound, the yo-yo rotates with angular velocity  $\omega_0$ . What is the minimum value for  $\omega_0$  required for the yo-yo to climb a distance  $l = 0.8\text{m}$  up the string (no slipping)?

**QUESTION 10**

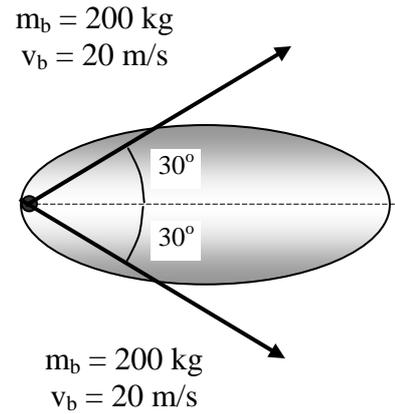
[Marks 20]

A ferry on a river is at rest relative to the water. A passenger spots a crocodile at rest in the water  $\ell = 8.0\text{m}$  from the front of the ferry. In response, the 50 passengers on board run to the far end of the ferry, trying to get as far from the crocodile as possible. The ferry has a mass  $M = 3500\text{kg}$  and length  $L = 30\text{m}$ ; its centre of mass lies at the centre of the ferry. The average mass of a passenger is  $75\text{kg}$ . Assume that originally the passengers were evenly distributed around the ferry and after the scare they are all a distance  $L/2$  from the centre. See the figure.



- How far from the crocodile is the ferry ( $d$  in the figure) after the passengers reach the end? [Hint: Consider the centre of mass of the ferry and passengers.]

- (b) What is the speed of the ferry (relative to the water) after the passengers reach the end?
- (c) The engine of the ferry will not start. In desperation, the passengers decide to construct a couple of catapults and throw their belongings into the river so that the ferry will recoil. The masses  $m_b$  and velocities  $v_b$  of the belongings and the angles at which they're thrown are shown in the figure. Calculate the final velocity of the ferry with respect to the water.



- (d) A person on the bank watches in amusement. What is the final speed of the ferry according to this observer if the river moves at a speed  $3\text{km/h}$  in the same direction as the ferry?